New Baryons in the $\Delta\eta$ and $\Delta\omega$ Channels

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Abstract

The decays of excited nonstrange baryons into the final states $\Delta\eta$ and $\Delta\omega$ are examined in a relativized quark pair creation model. The wavefunctions and parameters of the model are fixed by previous calculations of $N\pi$ and $N\pi\pi$, *etc.*, decays through various quasi-two body channels including $N\eta$ and $N\omega$. Our results show that the combination of thresholds just below the region of interest and the isospin selectivity of these channels should allow the discovery of several new baryons in such experiments.

I. INTRODUCTION

Quark models of baryon structure based on three effective quark degrees of freedom predict the existence of more states than have previously been seen in analyses of $N\pi$ elastic scattering. In particular, there are approximately nine 'missing' states predicted by quark potential models to lie in the first band of positive-parity excited states (which we define as states whose wavefunctions are predominantly $N = 2$ band when expanded in a harmonic oscillator basis). One of these states, a second $N\frac{3}{2}^+$ (P_{13}) resonance, may have been discovered in the coupled channel analysis of Manley and Saleski [1]. There remain six missing nucleon and two missing Δ states with model masses between approximately 1850 and 2050 MeV. There are also many undiscovered states predicted by these models which have wavefunctions which lie predominantly in the $N = 3$ and higher bands, the lightest of which are predicted to have negative parity [2,3].

Models of this kind, when combined with a model of the strong decays of baryon states [4], yield a simple explanation for the absence of the missing states [5,6,7,8] in analyses of $N\pi$ elastic scattering–they simply have weak $N\pi$ couplings and so contribute little to $N\pi$ scattering amplitudes in their partial waves. A simple solution is to produce these states electromagnetically with real photons or by electron scattering, and then look for their decays to final states other than $N\pi$ [9]. As part of the N^* program in Hall B at the Thomas Jefferson National Accelerator Facility (TJNAF), an experiment [10] will study $\gamma p \to p\pi^+\pi^-$, with analysis focusing on $\gamma p \to \Delta^{++}\pi^-$, $\gamma p \to \Delta^0\pi^+$ and $\gamma p \to p\rho^0$. Previous theoretical work [5,6,11,8] has shown that several of the missing states and many of the undiscovered states have sizeable couplings to these channels. Other experiments will focus on the decays of such states to $N\eta$ [12], $N\eta$ and $N\eta'$ [13], and $N\omega$ [14]. These channels offer the advantage of being isospin selective, in that only $I = \frac{1}{2}$ N^* resonances (as opposed to $I = \frac{3}{2}$ Δ^* resonances) can couple to these final states. Given the predicted near degeneracy of broad states in several partial waves in this region, this isospin selectivity should simplify what is likely to be a difficult analysis to extract information about these states.

Detection and analysis of the final states $\Delta(1232)\eta$ and $\Delta\omega$ in electromagnetic production from protons at TJNAF will be complicated by the increased particle multiplicity, and by the presence of an additional neutral particle in the final state resulting from the decays $\Delta^+ \to p\pi^0$, $n\pi^+$. For these experiments, it may be better to produce these final states from the neutron in the deuteron [15], reconstruct the $\Delta^0 \to p\pi^+$ charged particle decay, and use missing mass to identify the η or ω. At the AGS, the properties of the Crystal Barrel detector make it ideal for examining the process $\pi^-p \to n\pi^0\eta$. The final state in $\pi^+p \to p\pi^+\eta$ is more difficult to detect but can, in principle, also be seen using the Crystal Barrel [16]. Despite these detection difficulties, these channels also have the advantage of being isospin selective, and can in principle isolate the two missing Δ resonances and higher lying Δ states if they are present and are produced.

Another advantage of a $\Delta\eta$ experiment is that the threshold for this reaction lies just below the mass region where these states are predicted. The nominal $\Delta\eta$ threshold is at 1780 MeV; as we integrate over the lineshape of the Δ, the effective threshold is at $m_N + m_\pi + m_\eta \simeq 1630$ MeV. This is to be compared to mass predictions [2,3] for the lightest missing states of around 1800-1850 MeV $[\Delta\frac{1}{2}^+]_1$ and around 1950-2000 MeV for $[\Delta\frac{3}{2}^+]_4$ (which would be a first P_{31} and a fourth P_{33} state in $N\pi$, respectively). It is generally true (in

decay models and in experiment) that once the energy available for a decay increases beyond the region where the phase space has initially become appreciable, the decay amplitudes tend to decrease rapidly as the three momentum available to the final particles increases and the wavefunction overlaps diminish.

Here we provide predictions for the decay amplitudes into the final states $\Delta\eta$ and $\Delta\omega$ of all states (missing and seen in $N\pi$) with wavefunctions predominantly in the $N = 1$ and $N = 2$ bands, and also for several low-lying states in higher bands, using the relativized model of baryon decays based on the ${}^3P_0$ pair creation model of Refs. [7] and [11]. Decays into the $\Delta\eta$ channel have been previously considered by Bijker, Iachello and Leviatan in Ref. [8], within an algebraic model of the spectrum and wavefunctions, using pointlike emitted mesons.

In the present calculation model parameters are taken from our previous work and not adjusted. Wavefunctions are taken from the relativized model of Ref. [3], which describes all of the states considered here in a consistent picture. In order to be in accord with the Particle Data Group (PDG) [17] conventional definitions of decay widths, we have determined the decay momentum using the central value of the PDG quoted mass for resonances seen in $N\pi$, and the predicted mass from Ref. [3] for missing and undiscovered states. We have also integrated over the lineshape of the final Δ baryon, with the final phase space as prescribed in the meson decay calculation of Ref. [18]; for details of this procedure see Eq. (8) of Ref. [11] (note that we do not integrate over the narrow [8 MeV width] ω lineshape). As a consequence there are states below the nominal thresholds which have non-zero decay amplitudes.

In keeping with the convention of Ref. [11], the phases of the amplitudes are determined as follows. We quote the product $A^{X\dagger}_{\Delta M}A^X_{N\pi}/|A^X_{N\pi}|$ of the predicted decay amplitude for $X \to \Delta M$ (where M is either η or ω) and the phase of the decay amplitude for $X \to N\pi$, the latter being unobservable in $N\pi$ elastic scattering (note factors of $+i$, conventionally suppressed in quoting amplitudes for decays of negative parity baryons to NM or $N\gamma$, where M has negative parity, do not affect this product). This eliminates problems with (unphysical) sign conventions for wavefunctions, and the relative signs of these products are then predictions for the (physically significant) relative phases of the contributions of states X in the process $N\pi \to X \to \Delta M$. Since the missing and undiscovered states may have small $N\pi$ couplings it may be useful to find the relative signs of the contributions of states X in the process $N\gamma \to X \to \Delta\eta$. As the photocouplings of Ref. [19] are also quoted inclusive of the $N\pi$ sign, $A^{X\dagger}_{N\gamma}A^X_{N\pi}/|A^X_{N\pi}|$, then simply multiplying the quoted photocouplings by the amplitudes quoted here will yield the relative phases of the contributions of states X in $N\gamma \to X \to \Delta M$.

We note that we have chosen the meson wave flavor functions as

$$\begin{aligned}
\eta &= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(u\overline{u} + d\overline{d}\right) - s\overline{s}\right] \\
\eta' &= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(u\overline{u} + d\overline{d}\right) + s\overline{s}\right] \\
\omega &= \frac{1}{\sqrt{2}}\left(u\overline{u} + d\overline{d}\right), \qquad (1)
\end{aligned}$$

i.e. we allow for ideal mixing between ω and $\phi = s\overline{s}$, and an η–η' mixing angle of $\theta_P = -9.7\,^\circ$.

II. RESULTS AND DISCUSSION

Our results are given in Tables I to IV, where we list the model state, its assignment (if any) to a resonance from the analyses, and its decay amplitudes into the $\Delta\eta$ and $\Delta\omega$ channels. The predictions for the $N\pi$ decay amplitudes for each state [7] and values for these amplitudes extracted from the PDG [17] are also included for ease of identification of missing resonances. All theoretical amplitudes are given with upper and lower limits, along with the central value, in order to convey the uncertainty in our results due to the uncertainty in the resonance's mass. These correspond to our predictions for the amplitudes for a resonance whose mass is set to the upper and lower limits, and to the central value, of the experimentally determined mass. For states as yet unseen in the analyses of the data, we have adopted a 'standard' uncertainty in the mass of 150 MeV and used the model predictions for the state's mass for the central value. If a state below the effective threshold has been omitted from a table it is because our predictions for all of its amplitudes are zero.

For completeness we have also calculated decays to the $\Delta\eta'$ channel, and find that all of the amplitudes for the states considered here are small. This is primarily due to the high effective (nominal) threshold of roughly 2040 (2190) MeV. We do not record these amplitudes here.

A. $\Delta\eta$ decays

The results for this channel are shown in Tables I and II. Amplitudes for the lighter states are predictably small due to the effective (nominal) threshold of 1638 (1780) MeV, with some notable exceptions. In Table I, the largest amplitudes are those of the $\Delta(2000)F_{35}$ state, which is a two-star state [17]. This channel thus offers a very good opportunity for confirmation of this state. In addition, it should be possible to detect the missing fourth P_{33} state $[\Delta\frac{3}{2}^+]_4(1985)$, and to confirm the first P_{31} state seen in the multichannel analysis of Ref. [1], as their couplings to this channel are appreciable.

In table II, we see that it may be possible to confirm the one-star states $\Delta(1940)D_{35}$ and $\Delta(2390)F_{37}$ in a $\Delta\eta$ experiment. Our results predict that the model states $[\Delta\frac{3}{2}^-]_3(2145)$, $[\Delta\frac{5}{2}^-]_2(2165)$, $[\Delta\frac{7}{2}^-]_1(2230)$, $[\Delta\frac{7}{2}^-]_2(2295)$ and $[\Delta\frac{9}{2}^+]_2(2505)$ offer the best opportunities for discovery in this channel.

B. $\Delta\omega$ decays

The high effective (nominal) threshold of approximately 1860 (2010) MeV precludes sizeable couplings of states with wavefunctions predominantly below the $N = 3$ band to the $\Delta\omega$ channel (see Table III), although there are some states with amplitudes which grow rapidly away from the threshold, and so will couple if the actual mass is larger than the nominal mass. Key examples are the missing $[\Delta\frac{3}{2}^+]_4(1985)$ and the two-star $\Delta(2000)F_{35}$. However, there are several of the more highly excited states considered here in Table IV with appreciable couplings to this channel. It may be possible to confirm the weak states $\Delta(2150)S_{31}$ (one star), $\Delta(2400)G_{39}$ (two stars), and $\Delta(2390)F_{37}$ (one star) in this channel,

or perhaps even the very highly excited states $\Delta(2750)I_{3\,13}$ and $\Delta(2950)K_{3\,15}$ (both two-star states). Our results predict that a $\Delta\omega$ experiment may also be able to discover several predicted states, the most interesting of which are $\Delta[\frac{3}{2}^-]_3(2145)$, the two states $[\Delta\frac{5}{2}^-]_2(2165)$ and $[\Delta\frac{5}{2}^-]_3(2265)$, the two states $[\Delta\frac{7}{2}^-]_1(2230)$and $[\Delta\frac{7}{2}^-]_2(2295)$, and $[\Delta\frac{9}{2}^+]_2(2505)$.

C. Conclusions

Our results show that it should be possible to discover in a $\Delta\eta$ experiment one of the two low-lying states missing from previous $N\pi$ analyses, and to confirm the possible discovery of another. The fact that these are the first states in their partial waves to couple to this channel will make their extraction from an analysis less complicated than in final states with lower thresholds. There are also two weakly established states in the 1900–2000 MeV range which may be confirmed in such an experiment, and several higher mass predicted states which couple appreciably to this channel.

Amplitudes for all states to couple to the $\Delta\eta'$ channel are small, largely due to the high threshold. This is not true of the $\Delta\omega$ channel. Our results show that above 2150 MeV such an experiment may allow confirmation of several weakly established states and the discovery of a substantial number of predicted states.

Reconstruction of the $\Delta\eta$ and $\Delta\omega$ final states will be difficult due to the final state particle multiplicity and the presence of neutral particles in electromagnetic production from the proton. However it may be that the extraction of information about these important new baryon states from an analysis of the results of such an experiment is considerably less complicated than using channels with fewer final state particles. This indicates that it is worthwhile to consider developing such experiments.

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TABLES

TABLE I. Results for Δ states in the $N=1$ and $N=2$ bands in the $\Delta\eta$ channel. $N\pi$ amplitudes from Ref. [7] are included to explain our assignments of the model states to resonances. Notation for model states is $[J^P]_n$(mass[MeV]), where J^P is the spin/parity of the state and n its principal quantum number. The first row gives our model results, while the second row lists the $N\pi$ amplitudes from the partial-wave analyses, as well as the Particle Data Group (PDG) name for the state, its $N\pi$ partial wave, and its PDG star rating. Light states with zero amplitudes are omitted from the table.

model state $N\pi$ state/rating	$N\pi$	$\Delta\eta$	$\Delta\eta$	$\sqrt{\Gamma^{\rm tot}_{\Delta\eta}}$
		s	d	
$[\Delta\frac{3}{2}^-]_1(1620)$	4.9 ± 0.7	$1.1\ ^{+3.2}_{-1.1}$	$0.0\ ^{+0.3}_{-0.0}$	$1.1\ ^{+3.2}_{-1.1}$
$\Delta(1700)D_{33}$****	6.5 ± 2.0			
		p		
$[\Delta\frac{1}{2}^+]_1(1835)$	$3.9\ ^{+0.4}_{-0.7}$	$3.2\ ^{+4.1}_{-3.1}$		$3.2\ ^{+4.1}_{-3.1}$
$\Delta(1740)P_{31}$ [a]	4.9 ± 1.3			
$[\Delta\frac{1}{2}^+]_2(1875)$	9.4 ± 0.4	-2.9 ± 0.7		2.9 ± 0.7
$\Delta(1910)P_{31}$****	6.6 ± 1.6			
		p	f	
$[\Delta\frac{3}{2}^+]_2(1795)$	8.7 ± 0.2	$0.0\ ^{+0.3}_{-0.0}$	0.0 ± 0.0	$0.0\ ^{+0.3}_{-0.0}$
$\Delta(1600)P_{33}$***	7.6 ± 2.3			
$[\Delta\frac{3}{2}^+]_3(1915)$	4.2 ± 0.3	-3.3 ± 0.9	0.7 ± 0.4	3.4 ± 0.9
$\Delta(1920)P_{33}$***	7.7 ± 2.3			
$[\Delta\frac{3}{2}^+]_4(1985)$	$3.3\ ^{+0.8}_{-1.1}$	$-4.2\ ^{+2.4}_{-1.7}$	$-0.7\ ^{+0.6}_{-1.2}$	$4.3\ ^{+1.9}_{-2.5}$
		p	f	
$[\Delta\frac{5}{2}^+]_1(1910)$	3.4 ± 0.2	-0.5 ± 0.1	0.6 ± 0.3	0.8 ± 0.3
$\Delta(1750)F_{35}$ [b]	2.0 ± 0.8			
$\Delta(1905)F_{35}$****	5.5 ± 2.7			
$[\Delta\frac{5}{2}^+]_2(1990)$	1.2 ± 0.4	$-7.0\ ^{+5.1}_{-2.9}$	$0.3\ ^{+0.8}_{-0.3}$	$7.0\ ^{+2.9}_{-5.1}$
$\Delta(2000)F_{35}$**	5.3 ± 2.3			
		f	h	
$[\Delta\frac{7}{2}^+]_1(1940)]$	7.1 ± 0.1	0.9 ± 0.1	0.0 ± 0.0	0.9 ± 0.1
$\Delta(1950)F_{37}$****	9.8 ± 2.7			

a First P_{31} state found in Ref. [1].
b Ref. [1] finds two F_{35} states; this one and $\Delta(1905)F_{35}$.

TABLE II. Results in the $\Delta\eta$ channel for the lightest few negative-parity Δ resonances of each J in the N=3 band, and for the lightest few Δ resonances for J^P values which first appear in the N=4, 5 and 6 bands. Notation as in Table I.

model state $N\pi$ state/rating	$N\pi$	$\Delta\eta$	$\Delta\eta$	$\sqrt{\Gamma^{\rm tot}_{\Delta\eta}}$
		d		
$[\Delta\frac{1}{2}^-]_2(2035)$	1.2 ± 0.2	$1.8\ ^{+0.9}_{-0.7}$		$1.8\ ^{+0.9}_{-0.7}$
$\Delta\frac{1}{2}^-(1900)S_{31}$***	4.1 ± 2.2			
$[\Delta\frac{1}{2}^-]_3(2140)$	$3.1\ ^{+0.4}_{-1.1}$	$-2.4\ ^{+1.0}_{-.6}$		$2.4\ ^{+0.6}_{-1.0}$
$\Delta\frac{1}{2}^-(2150)S_{31}$*	4.0 ± 1.5			
		s	d	
$[\Delta\frac{3}{2}^-]_2(2080)$	2.1 ± 0.1	2.4 ± 0.6	$1.9\ ^{+1.9}_{-1.6}$	$3.1\ ^{+1.8}_{-1.3}$
$\Delta\frac{3}{2}^-(1940)D_{33}$*	3.2 ± 1.4			
$[\Delta\frac{3}{2}^-]_3(2145)$	$2.2\ ^{+0.1}_{-0.3}$	$-1.9\ ^{+0.4}_{-1.2}$	$3.3\ ^{+0.9}_{-1.5}$	3.8 ± 1.4
		d	g	
$[\Delta\frac{5}{2}^-]_1(2155)$	5.2 ± 0.0	1.1 ± 0.3	-0.1 ± 0.0	1.1 ± 0.3
$\Delta\frac{5}{2}^-(1930)D_{35}$***	5.0 ± 2.3			
$[\Delta\frac{5}{2}^-]_2(2165)$	0.6 ± 0.1	$3.7\ ^{+0.9}_{-1.6}$	$1.3\ ^{+1.2}_{-0.9}$	$3.9\ ^{+1.3}_{-1.8}$
$[\Delta\frac{5}{2}^-]_3(2265)$	2.4 ± 0.4	-2.7 ± 0.2	1.2 ± 0.4	2.9 ± 0.4
$[\Delta\frac{5}{2}^-]_4(2325)$	0.1 ± 0.0	$-2.4\ ^{+0.4}_{-0.1}$	$1.1\ ^{+0.7}_{-0.5}$	$2.6\ ^{+0.5}_{-0.6}$
$[\Delta\frac{7}{2}^-]_1(2230)$	2.1 ± 0.6	$3.8\ ^{+0.6}_{-1.5}$	$1.2\ ^{+1.0}_{-0.8}$	$4.0\ ^{+0.9}_{-1.7}$
$[\Delta\frac{7}{2}^-]_2(2295)$	1.8 ± 0.4	$-4.0\ ^{+1.0}_{-0.3}$	$1.5\ ^{+0.9}_{-0.8}$	$4.2\ ^{+0.6}_{-1.1}$
		g	i	
$[\Delta\frac{9}{2}^-]_1(2295)$	4.8 ± 1.3	$2.2\ ^{+2.1}_{-1.2}$	0.0 ± 0.0	$2.2\ ^{+2.1}_{-1.2}$
$\Delta\frac{9}{2}^-(2400)G_{39}$**	4.1 ± 2.1			
		f	h	
$[\Delta\frac{7}{2}^+]_2(2370)$	$1.5\ ^{+0.6}_{-0.9}$	$2.7\ ^{+0.4}_{-0.6}$	0.0 ± 0.0	$2.7\ ^{+0.4}_{-0.6}$
$\Delta\frac{7}{2}^+(2390)F_{37}$*	4.9 ± 2.0			
$[\Delta\frac{7}{2}^+]_3(2460)$	$1.1\ ^{+0.0}_{-0.1}$	-1.6 ± 0.4	$1.0\ ^{+0.9}_{-0.5}$	$1.9\ ^{+0.9}_{-0.6}$
$[\Delta\frac{9}{2}^+]_1(2420)$	1.2 ± 0.4	-0.2 ± 0.1	$0.7\ ^{+0.7}_{-0.4}$	$0.7\ ^{+0.7}_{-0.4}$
$\Delta\frac{9}{2}^+(2300)H_{39}$**	5.1 ± 2.2			
$[\Delta\frac{9}{2}^+]_2(2505)$	0.4 ± 0.1	-3.3 ± 0.7	$0.3\ ^{+0.3}_{-0.1}$	3.3 ± 0.8
		h	j	
$[\Delta\frac{11}{2}^+]_1(2450)$	2.9 ± 0.7	$1.0\ ^{+0.7}_{-0.4}$	0.0 ± 0.0	$1.0\ ^{+0.7}_{-0.4}$
$\Delta\frac{11}{2}^+(2420)H_{3\ 11}$****	6.7 ± 2.8			
$[\Delta\frac{13}{2}^+]_1(2880)$	0.8 ± 0.2	0.0 ± 0.0	$1.3\ ^{+0.8}_{-0.6}$	$1.3\ ^{+0.8}_{-0.6}$
$[\Delta\frac{13}{2}^+]_2(2955)$	0.2 ± 0.1	$-2.1\ ^{+0.4}_{-0.2}$	0.3 ± 0.1	$2.1\ ^{+0.2}_{-0.4}$
		i	k	
$[\Delta\frac{13}{2}^-]_1(2750)$	2.2 ± 0.4	$1.6\ ^{+0.7}_{-0.5}$	0.0 ± 0.0	$1.6\ ^{+0.7}_{-0.5}$
$\Delta\frac{13}{2}^-(2750)I_{3\ 13}$**	3.7 ± 1.5			
		j	l	
$[\Delta\frac{15}{2}^+]_1(2920)$	1.6 ± 0.3	1.4 ± 0.5	0.0 ± 0.0	1.4 ± 0.5
$\Delta\frac{15}{2}^+(2950)K_{3\ 15}$**	3.6 ± 1.5			

$[\Delta\frac{15}{2}^+]_2(3085)$	0.4 ± 0.1	0.4 ± 0.2	0.0 ± 0.0	0.4 ± 0.2

TABLE III. Results for Δ states in the $N = 1$ and $N = 2$ bands in the $\Delta\omega$ channel. Notation as in Table I.

model state $N\pi$ state/rating	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\sqrt{\Gamma^{\rm tot}_{\Delta\omega}}$
	$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{5}{2}}$				
$[\Delta\frac{1}{2}^+]_1(1835)$	$0.0\ ^{+1.1}_{-0.0}$	$0.0\ ^{+0.0}_{-2.2}$	0.0 ± 0.0				$0.0\ ^{+2.2}_{-0.0}$
$\Delta(1740)P_{31}$ [a]							
$[\Delta\frac{1}{2}^+]_2(1875)$	$0.1\ ^{+0.2}_{-0.1}$	0.0 ± 0.1	0.0 ± 0.0				$0.1\ ^{+0.2}_{-0.1}$
$\Delta(1910)P_{31}$****							
	$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{5}{2}}$				
$[\Delta\frac{3}{2}^+]_3(1915)$	0.0 ± 0.0	$-\ 0.1\ ^{+0.1}_{-0.3}$	0.0 ± 0.0	$-\ 0.1\ ^{+0.1}_{-0.2}$	0.0 ± 0.0		$0.1\ ^{+0.3}_{-0.1}$
$\Delta(1920)P_{33}$***							
$[\Delta\frac{3}{2}^+]_4(1985)$	$0.8\ ^{+3.8}_{-0.8}$	$0.3\ ^{+1.4}_{-0.3}$	$0.1\ ^{+1.3}_{-0.1}$	$0.0\ ^{+0.0}_{-0.1}$	$-\ 0.1\ ^{+0.1}_{-0.8}$		$0.9\ ^{+4.3}_{-0.9}$
	$f_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{3}{2}}$	$p_{\frac{5}{2}}$	$f_{\frac{5}{2}}$	$h_{\frac{5}{2}}$	
$[\Delta\frac{5}{2}^+]_1(1910)$	0.0 ± 0.0	$-\ 0.1\ ^{+0.1}_{-0.3}$	0.0 ± 0.0	$-\ 0.1\ ^{+0.1}_{-0.2}$	0.0 ± 0.0	0.0 ± 0.0	$0.1\ ^{+0.3}_{-0.1}$
$\Delta(1750)F_{35}$ [b]							
$\Delta(1905)F_{35}$****							
$[\Delta\frac{5}{2}^+]_2(1990)$	$0.0\ ^{+0.0}_{-0.5}$	$0.8\ ^{+3.6}_{-0.8}$	$0.1\ ^{+1.5}_{-0.1}$	$-1.3\ ^{+1.3}_{-5.6}$	$-\ 0.1\ ^{+0.1}_{-2.4}$	0.0 ± 0.0	$1.5\ ^{+7.2}_{-1.5}$
$\Delta(2000)F_{35}$**							
	$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	$p_{\frac{5}{2}}$	$f_{\frac{5}{2}}$	$h_{\frac{5}{2}}$	
$[\Delta\frac{7}{2}^+]_1(1940)]$	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	-1.0 ± 0.2	0.0 ± 0.0	0.0 ± 0.0	1.0 ± 0.3
$\Delta(1950)F_{37}$****							

a First P_{31} state found in Ref. [1].
b Ref. [1] finds two F_{35} states; this one and $\Delta(1905)F_{35}$.

TABLE IV. Results in the $\Delta\omega$ channel for the lightest few negative-parity Δ resonances of each J in the N=3 band, and for the lightest few Δ resonances for J^P values which first appear in the N=4, 5 and 6 bands. Notation as in Table I.

model state $N\pi$ state/rating	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\Delta\omega$	$\sqrt{\Gamma^{\rm tot}_{\Delta\omega}}$
	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$d_{\frac{5}{2}}$				
$[\Delta\frac{1}{2}^-]_2(2035)$	$-0.2^{+0.2}_{-0.6}$	$0.0^{+0.0}_{-0.2}$	0.0 ± 0.0				$0.2^{+0.7}_{-0.2}$
$\Delta\frac{1}{2}^-(1900)S_{31}$***							
$[\Delta\frac{1}{2}^-]_3(2140)$	-2.1 ± 0.7	0.1 ± 0.0	$5.7^{+5.8}_{-4.9}$				$6.1^{+5.8}_{-4.4}$
$\Delta\frac{1}{2}^-(2150)S_{31}$*							
	$d_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{5}{2}}$			
$[\Delta\frac{3}{2}^-]_2(2080)$	$0.1^{+1.4}_{-0.1}$	$-0.1^{+0.1}_{-2.1}$	0.0 ± 0.0	0.0 ± 0.0			$0.1^{+2.5}_{-0.1}$
$\Delta\frac{3}{2}^-(1940)D_{33}$*							
$[\Delta\frac{3}{2}^-]_3(2145)$	6 ± 0.6	0.2 ± 0.3	$-4.2^{+3.7}_{-4.5}$	0.0 ± 0.0			$4.2^{+4.5}_{-3.7}$
	$d_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	$s_{\frac{5}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{5}{2}}$	
$[\Delta\frac{5}{2}^-]_1(2155)$	0.0 ± 0.1	0.0 ± 0.1	0.0 ± 0.0	$-1.0^{+0.7}_{-1.4}$	-0.1 ± 0.0	0.0 ± 0.0	$1.0^{+1.4}_{-0.7}$
$\Delta\frac{5}{2}^-(1930)D_{35}$***							
$[\Delta\frac{5}{2}^-]_2(2165)$	$-1.0^{+0.9}_{-1.0}$	$-1.0^{+0.9}_{-1.0}$	$-0.5^{+0.5}_{-1.5}$	-3.3 ± 0.8	$-1.8^{+1.5}_{-1.6}$	0.0 ± 0.0	$4.0^{+2.3}_{-1.5}$
$[\Delta\frac{5}{2}^-]_3(2265)$	1.7 ± 0.5	-0.7 ± 0.2	0.3 ± 0.2	$-0.5^{+0.1}_{-0.3}$	$-3.0^{+0.9}_{-0.7}$	$-2.4^{+1.3}_{-1.9}$	$4.3^{+1.9}_{-1.5}$
$[\Delta\frac{5}{2}^-]_4(2325)$	-0.2 ± 0.1	-1.5 ± 0.7	$-0.2^{+0.2}_{-0.3}$	$1.0^{+0.7}_{-0.2}$	0.2 ± 0.1	$-1.1^{+0.8}_{-1.6}$	$2.1^{+1.8}_{-1.0}$
	$g_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{5}{2}}$	$i_{\frac{5}{2}}$	
$[\Delta\frac{7}{2}^-]_1(2230)$	$0.1^{+0.2}_{-0.1}$	-0.5 ± 0.3	$0.3^{+0.6}_{-0.3}$	-4.1 ± 2.2	$-1.4^{+1.1}_{-2.4}$	0.0 ± 0.0	$4.4^{+3.0}_{-2.4}$
$[\Delta\frac{7}{2}^-]_2(2295)$	$0.1^{+0.2}_{-0.1}$	-0.5 ± 0.3	$0.3^{+0.6}_{-0.3}$	-4.1 ± 2.2	$-1.4^{+1.1}_{-2.4}$	0.0 ± 0.0	$4.4^{+3.0}_{-2.4}$
	$g_{\frac{1}{2}}$	$g_{\frac{3}{2}}$	$i_{\frac{3}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{5}{2}}$	$i_{\frac{5}{2}}$	
$[\Delta\frac{9}{2}^-]_1(2295)$	$0.9^{+1.1}_{-0.7}$	$0.5^{+0.6}_{-0.4}$	0.0 ± 0.0	$-9.5^{+4.9}_{-1.4}$	$-1.9^{+1.5}_{-2.2}$	0.0 ± 0.0	$9.8^{+2.2}_{-5.1}$
$\Delta\frac{9}{2}^-(2400)G_{39}$**							
	$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	$p_{\frac{5}{2}}$	$f_{\frac{5}{2}}$	$h_{\frac{5}{2}}$	
$[\Delta\frac{7}{2}^+]_2(2370)$	1.4 ± 0.7	0.8 ± 0.4	0.0 ± 0.0	$-3.0^{+0.2}_{-1.4}$	$-3.1^{+1.7}_{-1.5}$	0.0 ± 0.0	$4.6^{+2.1}_{-1.4}$
$\Delta\frac{7}{2}^+(2390)F_{37}$*							
$[\Delta\frac{7}{2}^+]_3(2460)$	0.1 ± 0.0	-1.0 ± 0.6	$-0.2^{+0.2}_{-0.3}$	0.3 ± 0.0	$-1.6^{+0.9}_{-0.8}$	$-1.1^{+0.9}_{-1.4}$	$2.3^{+1.6}_{-1.3}$
	$h_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	$f_{\frac{5}{2}}$	$h_{\frac{5}{2}}$	$j_{\frac{5}{2}}$	
$[\Delta\frac{9}{2}^+]_1(2420)$	$0.2^{+0.4}_{-0.1}$	$-1.3^{+0.8}_{-1.2}$	$-0.1^{+0.1}_{-0.2}$	$-1.4^{+0.9}_{-1.3}$	$-0.3^{+0.3}_{-0.7}$	0.0 ± 0.0	$1.9^{+2.0}_{-1.3}$
$\Delta\frac{9}{2}^+(2300)H_{39}$**							
$[\Delta\frac{9}{2}^+]_2(2505)$	-0.4 ± 0.3	$2.4^{+0.5}_{-0.9}$	$1.0^{+0.9}_{-0.6}$	$-3.1^{+1.2}_{-0.7}$	$-1.3^{+0.8}_{-1.1}$	0.0 ± 0.0	$4.3^{+1.5}_{-1.8}$
	$h_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	$f_{\frac{5}{2}}$	$h_{\frac{5}{2}}$	$j_{\frac{5}{2}}$	
$[\Delta\frac{11}{2}^+]_1(2450)$	0.4 ± 0.3	0.2 ± 0.2	0.0 ± 0.0	$-5.1^{+2.2}_{-1.6}$	$0.8^{+0.6}_{-0.5}$	0.0 ± 0.0	$5.2^{+1.7}_{-2.3}$
$\Delta\frac{11}{2}^+(2420)H_{3\ 11}$****							
	$j_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	$h_{\frac{5}{2}}$	$j_{\frac{5}{2}}$	$l_{\frac{5}{2}}$	
$[\Delta\frac{13}{2}^+]_1(2880)$	$0.6^{+0.6}_{-0.3}$	-1.3 ± 0.5	$-0.3^{+0.2}_{-0.3}$	-1.5 ± 0.5	$-0.9^{+0.5}_{-0.9}$	0.0 ± 0.0	$2.3^{+1.3}_{-0.9}$
$[\Delta\frac{13}{2}^+]_2(2955)$	$-0.5^{+0.3}_{-0.4}$	1.6 ± 0.4	$1.2^{+1.0}_{-0.6}$	-1.9 ± 0.6	$-1.4^{+0.7}_{-1.1}$	0.0 ± 0.0	$3.2^{+1.5}_{-1.2}$

	$i_{\frac{1}{2}}$	$i_{\frac{3}{2}}$	$k_{\frac{3}{2}}$	$g_{\frac{5}{2}}$	$i_{\frac{5}{2}}$	$k_{\frac{5}{2}}$	
$[\Delta\frac{13}{2}^-]_1(2750)$	$0.6\ ^{+0.4}_{-0.2}$	0.3 ± 0.2	0.0 ± 0.0	$-4.3\ ^{+0.8}_{-1.0}$	$-1.1\ ^{+0.4}_{-0.6}$	0.0 ± 0.0	$4.5\ ^{+1.2}_{-0.9}$
$\Delta\frac{13}{2}^-(2750)I_{3\ 13}$**							
	$j_{\frac{1}{2}}$	$j_{\frac{3}{2}}$	$l_{\frac{3}{2}}$	$h_{\frac{5}{2}}$	$j_{\frac{5}{2}}$	$l_{\frac{5}{2}}$	
$[\Delta\frac{15}{2}^+]_1(2920)$	0.7 ± 0.3	0.3 ± 0.2	0.0 ± 0.0	-3.6 ± 0.8	$1.2\ ^{+0.6}_{-0.4}$	0.0 ± 0.0	3.9 ± 1.0
$\Delta\frac{15}{2}^+(2950)K_{3\ 15}$**							
$[\Delta\frac{15}{2}^+]_2(3085)$	0.2 ± 0.1	0.1 ± 0.1	0.0 ± 0.0	$-1.2\ ^{+0.3}_{-0.2}$	-0.4 ± 0.2	0.0 ± 0.0	1.3 ± 0.3